Limits on Steady Diffuse Mode Operation of the Cathode in Magnetoplasmadynamic Thrusters

V. V. Subramaniam† and K. S. Hoyer*
Ohio State University, Columbus, Ohio 43210
and
J. L. Lawless‡
Space Power, Inc., San Jose, California 95134

Electrode erosion in magnetoplasmadynamic thrusters is significantly dependent on whether the mode of operation is diffuse (i.e., the net current density is distributed over the electrode surface) or via spots. Since spot mode erosion rates are typically much higher than the diffuse mode erosion rates, the latter is more desirable from the standpoint of sustained steady-state operation. In this paper, the steady-state thermal response of the cathode in a magnetoplasmadynamic thruster is analyzed in order to determine the limits of diffuse mode operation. An energy balance at the cathode surface along with current conservation serve to simultaneously determine the surface temperature and the total sheath voltage drop at a given location on the electrode. It is found that two different limits to steady-state diffuse mode operation exist. One limit corresponds to an unsteady thermal runaway caused by excessive *electron bombardment*, in which regenerative heating leads to local melting of the electrode surface. The other limit corresponds to an operating regime where a steady state cannot exist because of increased ion bombardment. In this regime, a small increase in the net plasma current density results in a large sheath voltage drop. This causes an increase in local surface heating through ion bombardment, until a new state is reached, which is inherently unsteady and accompanied by plasma oscillations. These two limits found here may correspond to transitions from diffuse to prespot modes of electrode operation. These operating limits are also found to be influenced strongly by electrode and discharge parameters, as well as by external cooling.

Nomenclature

A = thermionic emission constant
C_e = absolute velocity of an electron
C_{ey} = component of the absolute velocity of an electron in the y direction, i.e., in the direction normal to the surface
E = electric field
E_c = magnitude of the y component of the electric field at the cathode surface
E_w = y component of the electric field at the sheath edge
e = electronic charge
f_e = electron velocity distribution function
h = heat transfer coefficient roughly representing overall heat transfer to an external coolant that is at a temperature T_{cool}
j_E = surface electron emission current density
j_e = current density of plasma electrons
j_i = current density of plasma ions
j_∞ = net plasma current density at a given axial location
K_t = thermal conductivity of cathode material
k = Boltzmann's constant
L = cathode length
m_e = mass of an electron
m_i = mass of an ion
n_e = electron number density
n_i = ion number density
n_∞ = charged particle number density at the edge of the collisionless sheath, at a given axial location
Q = net heat per unit area per unit time into the cathode surface, at a given axial location
T = local cathode surface temperature
T_{cool} = temperature of an external coolant
T_∞ = plasma electron temperature at a given axial location
V = voltage or potential
V_c = value of the voltage at the cathode surface at a given axial location, or the total voltage drop across the sheath
y = coordinate in the direction normal to the cathode surface
ε_i = ionization potential of the propellant gas
ϕ = electrode work function

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*Graduate Student, Department of Mechanical Engineering.
†DuPont Assistant Professor, Department of Mechanical Engineering. Member AIAA.
‡Senior Scientist and Manager of Advanced Concepts. Member AIAA.

I. Introduction

ELECTRODE erosion is of primary importance for the prediction of lifetimes of magnetoplasmadynamic (MPD) thrusters. Erosion processes depend on a complex coupling between plasma discharge characteristics, plasma-wall interactions, and electrode phenomena. In particular, erosion rates depend on whether the current conduction is through spots, or via a diffuse (distributed) mode. The diffuse mode is characterized by distributed current emission over the electrode surface, surface temperatures well below the material's melting temperature, and non-negligible plasma ion current. By contrast, spots are characterized by locally constricted or filamentary current conduction, surface temperatures at or above the material's melting temperature, and strong temperature-field emission. Spots are detrimental to the electrode material because of their high erosion rates.[1] Therefore, it is important to understand how and under what conditions they may be formed and exactly when diffuse mode behavior ends. Current understanding of this transition from diffuse to spot behavior is, at best, rather poor. In this paper, the steady-state diffuse mode at the cathode is examined theoretically. Limits on thruster operation in this mode are found, and an explanation for a possible transition to spot formation is given. Additionally, the analysis yields a means of estimating cathode surface temperature, which can be used to predict evaporative erosion rates for the cathode.

Earlier research on cathode processes has focused mainly on spot behavior and spot characteristics.[2-10] Although these works reveal the intricate and complex phenomena occurring in a cathode spot, little or no information is provided as to how spots are formed in the first place. Some authors have attempted to describe transition from diffuse to spot modes. Moizhes and Rybakov[11] have found a negative-slope region in the emission current voltage characteristic, which they attribute to transition to a spot mode. These authors attribute the transition to a thermal instability, whose physical meaning or origin is totally unclear. A thermal runaway mechanism in spots due to a positive feedback between Joule heating and a temperature dependent electrical conductivity has been proposed by Hantzsche.[12] In the same paper, however, the author shows that thermionic emission cools the surface and prevents this thermal runaway. It is important to note that Hantzsche assumes the sheath voltage drop to be constant and given.

By contrast to the cathode, transition at the anode appears to have been studied more extensively.[13-19] These works attribute spot formation to an initial local surface meltdown arising primarily from Joule heating at high current densities. However, another thermal runaway mechanism involving the much ignored sheath exists.[20] In this theory, electron emission from the electrode surface decreases the sheath potential drop, resulting in increased electron bombardment from the plasma. This results in a positive feedback between increased emission and heating due to electron bombardment, causing a thermal runaway. This may occur when the anode sheath voltage difference (defined here as anode potential minus the adjacent plasma potential) is negative and increasing. This thermal runaway mechanism arises before the well-known anode sheath reversal[21] and may explain formation of anode spots. In this paper, the same mechanism is shown to be also operative at the cathode.

A simple model of the cathode surface and its adjacent sheath is discussed in Sec. II. Results for a purely thermionically emitting cathode under a range of conditions corresponding to MPD thruster operation, are given in Sec. III. The physical meanings of the limits on steady-state diffuse mode operation are discussed in Sec. IV, and the effects of field-enhanced and temperature-field emission are discussed in Sec. V. Finally, a summary of this work along with its conclusions are presented in Sec. VI.

II. Simple Model

This section will focus on a thermal model of the cathode surface at steady state. The cathode surface is subjected to both heating and cooling (Fig. 1). Charged particles (i.e., electrons and ions) from the plasma bombard the surface and consequently heat it. The ions (which we will assume to be singly charged) also recombine with electrons at the electrode,

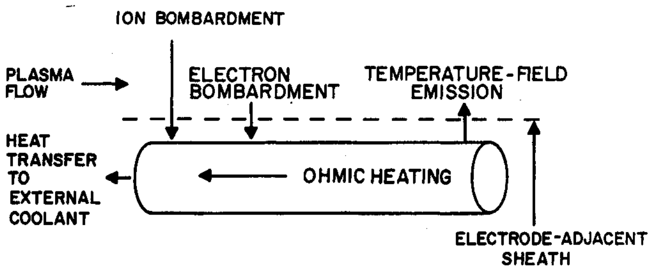

Fig. 1 Cathode along with the various heating and cooling mechanisms.

thereby heating the surface even further. Additional heating due to radiation from the plasma is also present and expected to be important,[22] but will be neglected here. The electrode may also emit electrons, which can result in heating (during field emission) or in cooling (during thermionic emission). During steady-state operation, it is likely that the electrode is cooled externally.[1,23] This is usually done by cooling the cathode at its base. Finally, the cathode surface may radiate away its heat. With these considerations, we may proceed to write the following for the net heat area per unit and per unit time entering the cathode surface:

$$Q = j_e\left[\phi + \frac{2kT_\infty}{e}\right] + j_i(V_c + \varepsilon_i - \phi) - j_E\left[\phi + \frac{2kT}{e}\right] - h(T - T_{\text{cool}}) \tag{1}$$

The quantity $h(T - T_{\text{cool}})$ represents a very rough model of the overall cooling, and h can be considered as equivalent to the electrode material thermal conductivity per unit length (i.e., K_t/L). At steady state, Q must be 0 and Eq. (1) determines the surface temperature T. Radiative heat transfer to the anode can be shown to be negligible and is, therefore, ignored here.

Consider next the sheath region immediately adjacent to the electrode surface. This region is typically of the order of tens of Debye lengths. The electron mean free path, however, is orders of magnitude larger.[22] Consequently, the sheath may be treated as a collisionless sheath. The electrons entering the sheath from the plasma are then described by the collisionless steady Boltzmann equation, which in one dimension is

$$C_{ey}\frac{\partial f_e}{\partial_y} + \frac{eE}{m_e}\frac{\partial f_e}{\partial C_{ey}} = 0 \tag{2}$$

Using the following transformations,

$$\xi = \frac{m_e C_e^2}{2kT_\infty} + \frac{eV}{kT_\infty}, \qquad \eta = y \tag{3}$$

along with $E = -\partial V/\partial y$, yields

$$\frac{\partial f_e}{\partial \eta} = 0, \qquad \text{or that} \qquad f_e = f_e(\xi) \quad \text{only} \tag{4}$$

With the condition that the electron distribution function be Maxwellian at the sheath edge (where V is taken to be 0), we obtain the following exact solution for the electron distribution function in the sheath:

$$f_e = n_e(m_e/2\pi kT_\infty)^{1/2}\exp[-m_e C_e^2/2kT_\infty]. \tag{5}$$

where

$$n_e = n_\infty \exp[-eV/kT_\infty] \tag{6}$$

with n_∞ being the electron number density at the edge of the sheath where V is taken to be 0. The average flux of electrons may then be calculated easily from Eqs. (5) and (6) by integrating f_e over dC_{ex} and dC_{ez} from $-\infty$ to ∞, and over dC_{ey} from 0 to ∞. This flux when evaluated at the wall and multiplied by e gives the electron current density at the wall:

$$j_e = en_\infty(kT_\infty/2m_e\pi)^{1/2}\exp[-eV_c/kT_\infty] \tag{7}$$

where $-V_c$ is the potential of the cathode with respect to the sheath edge (where $V = 0$), and j_e is taken to be positive in the direction away from the cathode.

The ions entering the sheath are assumed to be monoenergetic. The Bohm criterion for a stable monotonic sheath is, therefore, satisfied.[24] This condition is only slightly altered in the presence of electron emission from the surface.[25] More exact theories that provide the velocity distribution function of the ions entering the sheath from the collisional presheath exist.[26-28] However, for present purposes, the frequently employed monoenergetic assumption simplifies the problem a great deal. We may then write the ion current density at the surface as

$$j_i = en_\infty(kT_\infty/m_i)^{1/2} \tag{8}$$

where j_i is taken to be positive toward the cathode and quasineutrality ($n_i \approx n_e = n_\infty$) is assumed as the sheath edge. The sheath edge is a rather arbitrary definition since no such rigid demarcation exists in reality. When the ions are monoenergetic, the sheath edge has a clear meaning. However, even when the ion distribution function is not monoenergetic, the sheath edge can be defined as the location where the velocity of the ions reaches the value of the Bohm velocity $(kT\infty/m_i)^{1/2}$. In this paper, we will adopt this definition and take the data for the sheath potential at this location (i.e., $V = 0$ at the sheath edge). Now, if the surface emission current density denoted by j_E is taken to be positive toward the surface and the net plasma current density denoted by j_∞ is taken to be positive toward the cathode, we may write overall current conservation at steady state as

$$j_\infty = j_E + j_i - j_e \tag{9}$$

where, in general, j_E is a function of the local surface temperature T and the local electric field at the cathode surface E_c. Determination of E_c is discussed in Sec. V. Equation (9) determines the total cathode sheath voltage drop V_c. Given the parameters n_∞, j_∞, T_∞, ϕ, h, T_{cool} and the surface electric field E_c, Eqs. (1) and (9) represent two simultaneous equations in the two unknowns T and V_c. It must be pointed out that Crawford and Cannara mention overall current conservation in their work.[29] However, their regime of interest was at very low densities ($10^{14}\ m^{-3}$), and, hence, ion and electron currents from the plasma were negligible in comparison with the emission current. This is an important difference between their work and the present work.

A simple solution may be found for the case of pure thermionic emission (which is likely for refractory materials), where the surface emission current density depends only on T. For this case, the total sheath voltage drop may be explicitly obtained from Eq. (9):

$$V_c = -\frac{kT_\infty}{e}\,\ell n\left(\frac{j_i + j_E - j_\infty}{j_r}\right) \tag{10}$$

where $j_r = en_\infty(kT_\infty/2\pi m_e)^{1/2}$. Substituting Eq. (10) into Eq. (1) yields the following:

$$Q = \frac{2kj_E}{e}(T_\infty - T) + j_i\left(V_c + \varepsilon_i + \frac{2kT_\infty}{e}\right) - j_\infty\left(\phi + \frac{2kT_\infty}{e}\right) - h(T - T_{\text{cool}}) \tag{11}$$

Since at steady state Q must be zero, Eq. (11) represents an implicit equation for the local surface temperature T, where V_c is given by Eq. (10).

An interesting phenomenon is revealed by Eq. (11). The emission current density, which under thermionic conditions results in energy transport away from the cathode, appears as an input heating source. This can be understood upon close examination of Eq. (9) or (10). When the emission current density increases for a fixed plasma number density and plasma current density, the sheath voltage drop must decrease in order to conserve total current. This results in an increase in electron bombardment, which contributes to heating the surface. Thus, although thermionic emission cools the surface, its overall effect is to *heat* the surface via increased electron bombardment. It is, therefore, important to include the effects of the sheath when attempting to determine the surface temperature.

A simple model has been developed in this section, which considers a thermal balance of the cathode surface along with overall current conservation. The local surface temperature and overall sheath voltage drop can be determined from this. Results for a purely thermionically emitting cathode under typical MPD conditions are presented next.

III. Results for Pure Thermionic Emission

The governing equations presented in the preceding section are solved in this section for the case of pure thermionic emission (i.e., no field-enhanced or temperature-field emission). Temperatures and sheath voltage drops are calculated for typical MPD conditions.

For pure thermionic emission, the emission current density is given by

$$j_E = AT^2 \exp[-e\phi/kT] \tag{12}$$

The sheath voltage drop V_c can then be written in terms of T using Eq. (10). Given the parameters n_∞, j_∞, T_∞, ϕ, h, and T_{cool}, Eqs. (10-12) may be combined and solved for T. The sheath voltage drop can then be determined from Eq. (10). The emission coefficient may vary considerably for materials with oxide layers. Both A and ϕ may vary locally on a given surface, so that it is important to consider the sensitivity of final results to variations in these parameters. This is addressed later in this section.

A typical variation of the net heat into the cathode Q vs the surface temperature is displayed in Fig. 2. Two intersections with the horizontal axis are found, representing two possible steady-state solutions. The first (lower temperature) is a stable attractor and the second (higher temperature) is an unstable repeller. This can be seen quite easily by perturbing the solutions to either side and determining if the initial state is restored. The physical meaning of the stable point is clear. The incoming energy on the surface is exactly balanced by the outgoing energy. Furthermore, the surface

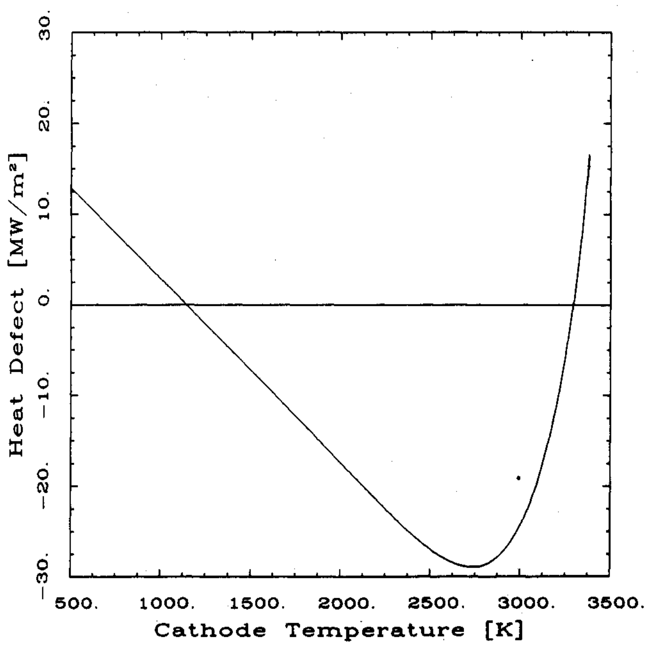

Fig. 2 Typical variation of the net heat per unit area per unit time into the cathode surface vs surface temperature. Variation predicted by Eqs. (10) and (11) in Sec. II.

can maintain this temperature for an indefinite period of time, even for small fluctuations in the discharge. The unstable point requires some explanation. Although an exact balance between incoming and outgoing energy is possible at this point, the surface cannot remain in this state indefinitely and is susceptible to change due to infinitesimal fluctuations. Physically, this state represents an operating regime where the temperature and, hence, the emission current density is high enough to lower the sheath voltage drop below a critical value. This causes excessive heating due to increased electron bombardment, resulting in a temperature rise. The subsequent increase in temperature causes an increase in current emission, which lowers the sheath voltage drop further. This positive feedback process then repeats itself until the surface is regeneratively heated up to the melting temperature. This effect is quantitatively described in Sec. IV.

Results from the present calculation are shown in Figs. 3–6. In these figures, the steady-state surface temperatures calculated from Eq. (11) are shown vs the net plasma current density j_∞, the charged particle number density at the sheath edge n_∞, the electrode work function ϕ, and the temperature of the external coolant T_{cool}. The heat transfer coefficient h is fixed at 20 kW/m²/K, the plasma electron temperature is taken to be 12,000 K, and $A = 3 \times 10^4$ A/m²/K². The value of h is obtained from an estimate of the thermal conductance through a tungsten cathode. All four plots display both the stable (lower temperature) and unstable (higher temperature) regimes. Figures 3 and 4 display the variation of T with j_∞ and n_∞, respectively, for $\phi = 2.63$ V and $T_{cool} = 500$ K. From Fig. 3, it can be seen that, for a given number density, the stable diffuse mode is limited both at low as well as high values of the current density. Similarly, for a given current density, there are lower and upper limits on the range of allowable number densities. At low current densities (n_∞ fixed) and at high number densities (j_∞ fixed), the stable and unstable solutions are seen to merge and disappear. Also, at the higher current densities (n_∞ fixed) and the lower number densities (j_∞ fixed), the stable diffuse operation is limited and only the unstable mode exists. It is interesting to note that, for a given number density, the steady-state, stable surface temperature does not vary significantly with the current density over a

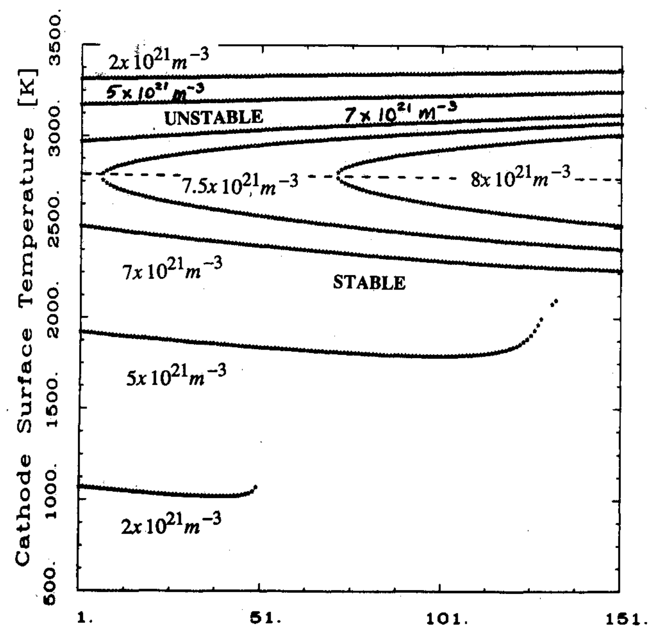

Fig. 3 Steady-state surface temperature as predicted by Eqs. (10) and (11) vs net plasma current density j_∞ for various values of n_∞: $h = 20$ kW/m²/K; $\phi = 2.63$ V; $T_{cool} = 500$ K; $A = 3 \times 10^4$ A/m²/K²; $T_\infty = 12000$ K.

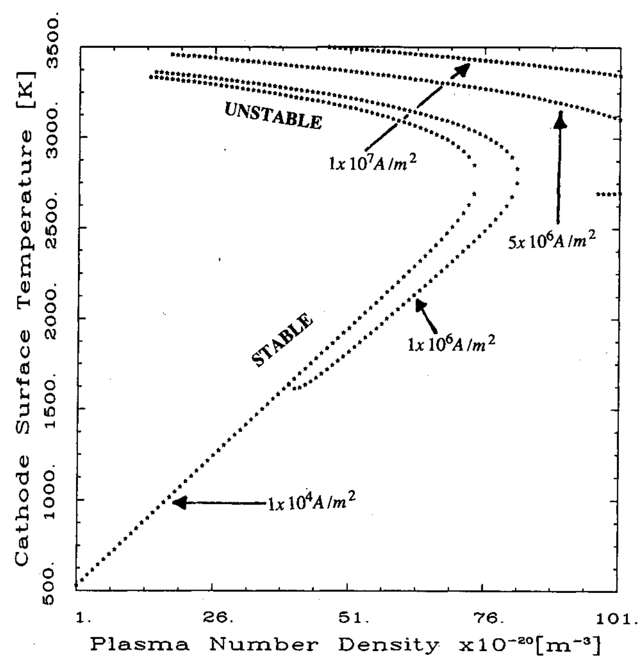

Fig. 4 Steady-state surface temperature as predicted by Eqs. (10) and (11) vs net plasma current density n_∞ for various values of j_∞: $h = 20$ kW/m²/K; $\phi = 2.63$ V; $T_{cool} = 500$ K; $A = 3 \times 10^4$ A/m²/K²; $T_\infty = 12000$ K.

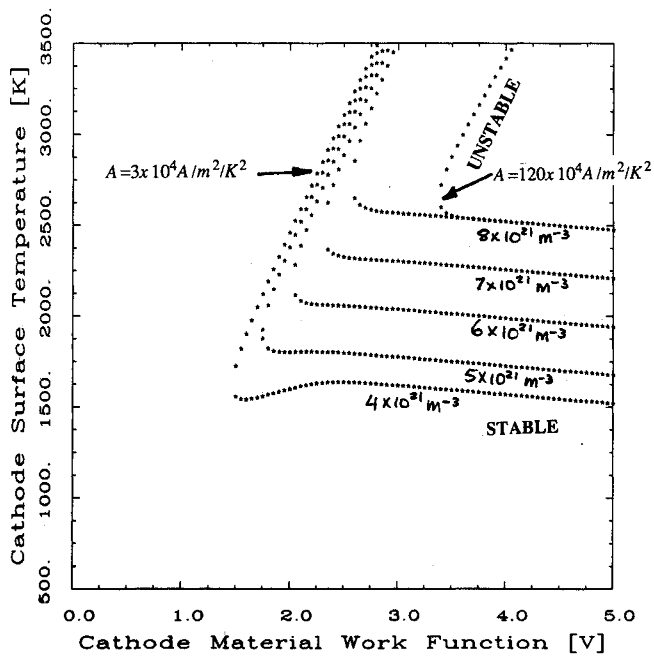

Fig. 5 Steady-state surface temperature as predicted by Eqs. (10) and (11) vs electrode work function for $j_\infty = 10^6$ A/m², $n_\infty = 8 \times 10^{21}$ m⁻³, $T_{cool} = 500$ K, $h = 20$ kW/m²/K, and $T_\infty = 12000$ K.

wide range (see Fig. 3). This is because in this region the ion current primarily determines the sheath voltage drop, which is then to a very good approximation a constant since the number density is fixed. Heating due to ion bombardment is then balanced by the external cooling. Therefore, inspection of Eq. (11) reveals that the surface temperature should not depend significantly on j_∞ in this region. By contrast, changes in n_∞ at a fixed j_∞ affect the stable operation drastically via increased electron and/or ion bombardment. This is reflected in Fig. 4. The variation of the surface temperatures with electrode work function is shown in Fig. 5 for $T_{cool} = 500$ K, $j_\infty = 10^6$ A/m², and $n_\infty = 8 \times 10^{21}$ m⁻³. The previously discussed stable and unstable regimes are again found to exist. The

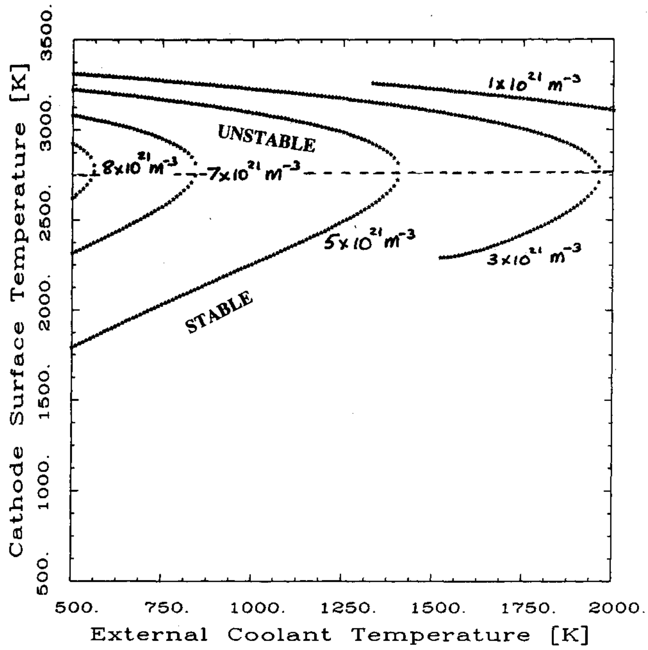

Fig. 6 Steady-state surface temperature as predicted by Eqs. (10) and (11) vs external coolant temperature T_{cool} for $\phi = 2.63$ V, $j_\infty = 10^6$ A/m², $h = 20$ kW/m²/K, A = 3 × 10⁴ A/m²/K², and $T_\infty = 12000$ K.

stable, steady-state surface temperature is almost invariant with respect to ϕ in this regime, again because the ion current principally determines the sheath drop and ion bombardment dominates. Also shown in this figure is the solution for a different value of the thermionic emission constant ($A = 120 \times 10^4$ A/m²/K²). The effect of a higher value of A is simply to shift the turning point (i.e., the point where stable and unstable solutions merge and then disappear) toward the higher work function side. Otherwise, the stable, steady-state surface temperature is practically the same as for a lower value of A since ion bombardment is still the dominant mechanism. Figure 6 displays the surface temperatures vs the external coolant temperature for $\phi = 2.63$ V, $j_\infty = 10^6$ A/m², $n_\infty = 5 \times 10^{21}$ m^{-3}, and $h = 20$ kW/m²/K. Both stable and unstable regimes are seen to exist and exhibit the features that have just been discussed.

This section has focused on results for a thermionically emitting cathode operating under typical MPD conditions. Stable and unstable regimes as well as limits on steady-state operation have been found. These limits are found to be influenced strongly by both discharge and electrode parameters. The next section will focus on a detailed discussion of the operating limits that have been found.

IV. Limits on the Steady Diffuse Mode

In this section, two different mechanisms that lead to local melting of the cathode surface are discussed. The first mechanism is an excessive heating due to electron bombardment that occurs when the sheath voltage drop falls below a critical value. A positive feedback between electron bombardment and surface electron emission results in this thermal runaway. The second mechanism is heating due to ion bombardment, which dominates as the sheath voltage drop increases sharply for a relatively small increase in the plasma current density. A steady state can no longer be maintained unless plasma discharge parameters change dramatically.

We consider the thermal runaway mechanism first. The value of the critical voltage drop at which the thermal runaway due to electron bombardment occurs can be obtained quite readily for the case of pure thermionic emission. For a given plasma current density and plasma number density at the sheath edge, Eq. (10) may be substituted into Eq. (11) and the resulting expression differentiated with respect to T to yield

$$\frac{\partial Q}{\partial T} = -h + \frac{2kj_E}{e}\left(-\frac{e\phi}{kT} - 3\right) + \frac{2kT_\infty j_E}{eT}\left(2 + \frac{e\phi}{kT}\right)\left(1 - \frac{j_i}{2j_e}\right) \tag{13}$$

For stability (in the Lyapunov sense), we require that $\partial Q/\partial T$ be less than zero. A *sufficient* condition for stability from Eq. (13) is

$$\frac{j_i}{j_e} \geq 2 \tag{14}$$

which, by using Eqs. (7) and (8), gives a condition on the sheath voltage drop V_c:

$$V_c \geq \frac{kT_\infty}{2e} \ln\left[\frac{2m_i}{\pi m_e}\right] \tag{15}$$

For argon and an electron temperature of 12,000 K, this yields an approximate value of 5.5 V for the critical sheath voltage drop. It is important to note that this is only a *sufficient* condition for stability. From Eq. (13), it can be seen that the exact value of the critical voltage drop depends on the electrode properties (work function and emission current), surface temperature, and external cooling h. This more general criterion on the total sheath voltage drop necessary to ensure stability of a given steady state is

$$V_c \geq \frac{kT_\infty}{2e} \ln\left[\left(\frac{m_i}{2\pi m_e}\right)^{1/2} \left(2 - \frac{T(3 + e\phi/kT)}{T_\infty(2 + e\phi/kT)} - \frac{ehT}{kj_E T_\infty(2 + e\phi/kT)}\right)\right] \tag{16}$$

Above the value given by the critical voltage drop, a steady state defined by Eqs. (10) and (11) (with $Q = 0$) can be maintained indefinitely, even for small fluctuations in n_∞ and j_∞. Below this value, the positive feedback between electron bombardment and surface electron emission leads to thermal runaway, and a stable steady state cannot be maintained. The surface subsequently melts locally. Alternatively, the condition $\partial Q/\partial T < 0$ may be interpreted as providing a constraint on the amount of cooling (i.e., the value of h) necessary for steady and stable operation. Equation (14) along with Eq. (9) may be rewritten, therefore, in the following form, yielding an important upper limit on the local surface temperature for diffuse operation:

$$j_E(T) = AT^2 \exp(-e\phi/kT) \leq j_\infty - j_e \tag{17}$$

Under MPD conditions, the critical temperatures predicted by Eq. (17) are well below the material melting temperature.

A second mechanism exists that can cause local melting of the cathode surface. For high j_∞ (given an n_∞) or low n_∞ (given a j_∞), it is possible that $j_\infty \to j_i + j_E$. When this occurs, the total sheath voltage drop increases sharply. This can be seen from Eq. (10), where $V_c \to \infty$ as $j_\infty \to j_i + j_E$. This has two important consequences. First, emitted electrons cause increased ionization outside the sheath since they gain large amounts of energy after traversing the sheath. Second, the ions gain substantial amounts of energy while falling through the sheath. These cause a rapid increase in ion bombardment with subsequent heating of the surface. This temperature rise necessarily results in increased surface electron emission.

Consequently, there arises a conflict between the emission current necessary for cooling the cathode surface (i.e., to balance heating by ion bombardment) and the emission current necessary to preserve overall current conservation at steady state. The increasing sheath drop also drastically reduces the electron current from the plasma necessary to counteract an increasing emission current and, hence, maintain a steady state. This leads to unsteady behavior in the sheath, thereby forcing the surface and the plasma to also become unsteady. In the meanwhile, the local surface temperature can rise due to ion bombardment until the surface has locally melted. Although the present theory becomes inapplicable at this point, it is capable of predicting this limit. An important phenomenon associated with this second mechanism, but not with the first, is the triggering of plasma oscillations.[30] When the sheath voltage drop rises and approaches energies equal to or greater than that of the first excited states of the propellant atoms, ionization levels can be enhanced in the presheath region. Also, while the thermionically emitted electrons relax their momenta relatively quickly due to elastic collisions with neutrals, their energy relaxation times are much longer due to infrequent collisions with plasma electrons. When the number density of thermionically emitted (beam) electrons approaches the local number density of plasma electrons, longitudinal oscillations can result.[30] This condition may be expressed quantitatively within the context of the present work as

$$n_e << 2.563 \times 10^{13} j_E^{2/5} V_C^{8/5} \tag{18}$$

where n_e is the number density of plasma electrons immediately adjacent to the sheath. This second mechanism, which is related to a high back-EMF (Electro Motor Force), may explain the onset phenomenon observed in the MPD thrusters.[31,32]

The two aforementioned mechanisms responsible for limiting the diffuse mode also cause local melting of the cathode surface. Local surface melting may be a precursor to spot formation. However, it must be pointed out that an intermediate stable state may exist where molten regions that are submicron in size (microspots[1]) are scattered over the cathode surface while most of the discharge appears diffuse. It is therefore possible that the two mechanisms found and discussed here correspond to transitions from diffuse to spot mode or another intermediate mode with microspots.

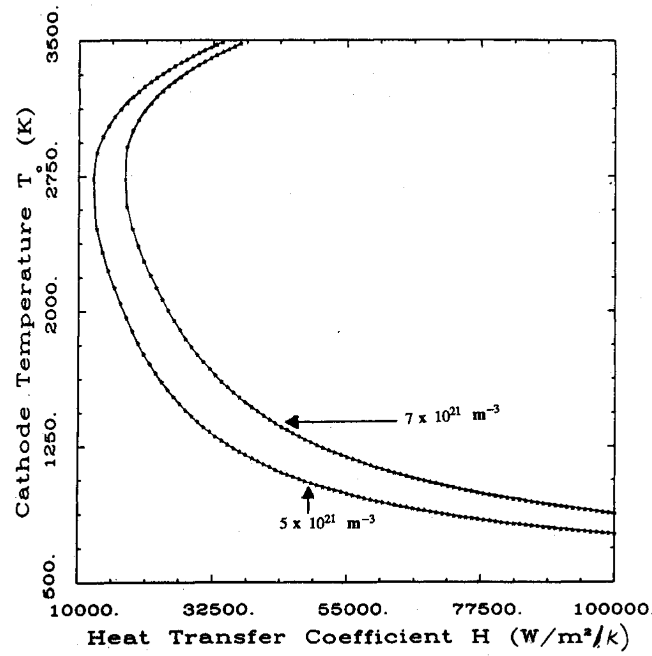

Fig. 8 Variation of the cathode surface temperature as predicted by Eqs. (10) and (11) vs the heat transfer coefficient h for a net current density of j_∞ = 10^6 A/m² and for two different number densities: T_∞ = 12,000 K; ϕ = 2.63 V; T_{cool} = 500 K.

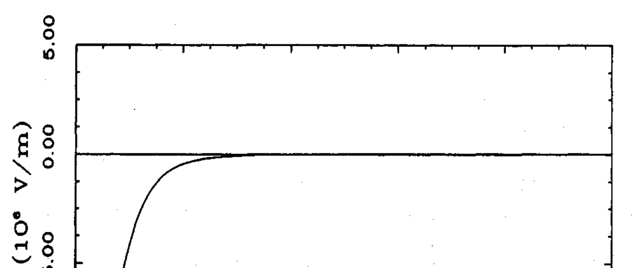

Fig. 7 Variation of the electric field in the steady, collisionless electrode-adjacent sheath vs vertical distance from the cathode surface: j_∞ = 10^6 A/m²; n_∞ = 5 × 10^{21} m⁻³; T_∞ = 12000 K; T_{cool} = 500 K; ϕ = 2.63 V; h = 20 kW/m²/K²; the calculated surface temperature is 1824 K; the value of the electric field at the cathode surface is 1.76 × 10^7 V/m. (Note that the electric field is negative because of the convention used here. The actual field is positive, i.e., pointing toward the surface.)

V. Effects of Field-Enhanced and Temperature-Field Emission

Thus far, we have considered a purely thermionically emitting cathode. In general, however, the emission current density depends both on the surface temperature and on the surface electric field.[33] In this section, the influence of this field-enhanced (also known as Schottky emission) and temperature-field (T-F) emission is evaluated and discussed.

The governing equations for the sheath and the cathode surface are described by the surface energy balance [Eq. (1)] and overall current conservation [Eq. (9)]. These equations are still applicable in the presence of an electric field at the surface of the cathode. However, a simple analytic solution cannot be obtained because the sheath voltage drop cannot be determined explicitly in terms of the surface temperature. This is because the emission current density in Eq. (9) depends on T and E_c, and E_c, in turn, depends on V_c. The required additional equation for E_c may be obtained from a solution of Poisson's equation in the sheath. An outline of this procedure is given by Prewett and Allen[25] for the case of a constant surface emission current density. Extension of their solution for the potential distribution in the sheath, for the case where the emission current density depends on both T and E_c, is straightforward and will not be discussed here. The additional equation for E_c for monoenergetic ions is the following implicit equation:

$$\frac{\varepsilon_o}{2}(E_c^2 - E_w^2) \approx n_\infty kT_\infty \left[\left(1 + \frac{2eV_c}{kT_\infty}\right)^{1/2} - 2 + \exp[-eV_c/kT_\infty]\right] - j_E(2m_eV_c/e)^{1/2} \tag{19}$$

where E_w is the electric field at the sheath edge where $V = 0$. In general, E_w is much smaller than E_c and may be ignored, and the emission current density j_E may be calculated from quantum-mechanical considerations. A general integral expression for j_E as a function of T and E_c is given by Murphy and Good.[33] For low values of E_c, j_E may be simply expressed by the well-known Schottky formula.[33] For the higher values of E_c, the emission is influenced by both E_c and T and is known as T-F emission. Thus, Eqs. (1), with $Q = 0$, (9), and (19) represent three coupled and nonlinear equations for the three unknowns T, V_c, and E_c. This system has been solved numerically. It is also possible to numerically integrate Poisson's equation to determine the variation of the potential and the electric field in the sheath. Figure 7 displays the computed variation of the electric field vs vertical distance from the surface for a particular value of j_∞ and n_∞. However, it must be pointed out that the resulting profiles represent the solution within the sheath only. A uniformly valid solution for the sheath, transition region, and the plasma beyond has to be obtained through matched asymptotic expansions.[26-28] Consequently, the solution of Poisson's equation given here represents only the inner solution. For the range of parameters considered in this paper, computed values of E_c are of the order of 10^7 V/m or less. Such values of the electric field produce a negligible increase in current emission at surface temperatures of interest to the MPD thruster. Consequently, the stable diffuse mode is well represented by pure thermionic emission.

VI. Discussion and Conclusions

The thermal response of the cathode in an MPD thruster has been studied under steady-state diffuse conditions. The electrode-adjacent sheath has been included for monoenergetic ions. The solution of Poisson's equation in the sheath, together with overall current conservation and an energy balance at the electrode surface, serve to determine the electric field at the surface, the surface temperature, and the total sheath voltage drop simultaneously and self-consistently. Two operating regimes are found. One is stable, while the other is an unstable thermal runaway caused by excessive *electron bombardment*. The thermal runaway leads to eventual local melting of the cathode surface and may be a precursor to spot formation. The stable steady state, on the other hand, is influenced strongly by discharge and electrode parameters. The values of n_∞, j_∞, and T_∞ used here have been obtained from an approximate nonequilibrium model of the electrode-adjacent flow.[22] The rate of cooling is found also to be extremely important. Varying values of h result in two operating regimes, similar to the results displayed in Fig. 6. Furthermore, the dependence of the surface temperature on h is quite significant. For $n_\infty = 5 \times 10^{21}$ m^{-3}, $j_\infty = 10^6$ A/m^2, $T_{\rm cool} = 500$ K, and $\phi = 2.63$ V, a 1% decrease in h results in about a 10% increase in the local surface temperature (see Fig. 8). Of course, for h larger ($\approx 10^5$ W/m^2/K) than the value used here, the variation is less dramatic but still non-negligible.

Two mechanisms are found to be responsible for destabilizing the stable, steady, diffuse mode. The first occurs at values of the sheath voltage drop *below* a critical value. Small sheath voltage drops lead to increased electron bombardment and subsequent thermal runaway if the cooling is insufficient. Such a condition can occur in the MPD thruster operating at near-onset total currents. Near onset, the current density is sharply peaked and large near the inlet and the exit regions while going through a minimum in the middle region (in the axial direction) of the thruster. The current density in the middle region of the thruster becomes smaller with increasing total current to the thruster, due to a high back-EMF,[31,32] which then leads to smaller sheath voltage drops. The second mechanism, due to excessive ion bombardment and accompanied by plasma oscillations, occurs at high values of the current density (for a given plasma number density) or for low values of the plasma number density (for a given current density). Here, high sheath voltage drops result in increased ion bombardment leading to increased surface electron emission. Since plasma electrons are repelled at high sheath potentials, overall current conservation given by Eq. (9) cannot be satisfied at steady state. The sheath is then no longer steady. The second mechanism may be inhibited if an increase in the sheath edge charged particle number density n_∞ occurs via increased ionization due to electrons emitted from the cathode surface. Clearly, this requires overall sheath voltage drops of the order of the propellant ionization potential. The occurrence of both mechanisms is strongly influenced by discharge parameters.

The simple model discussed in this paper represents a first attempt at relating the plasma discharge, the sheath, and the electrode. Although this model has revealed some important underlying physics, there are several deficiencies. First, the presheath has not been considered in detail. The presheath is important not only for determining the ion velocity distribution function, but also for asymptotically determining a uniformly valid potential distribution. This potential distribution will then yield the correct variation in the sheath when considered on the scale of the Debye length and will also yield the correct variation in the plasma when viewed on the macroscopic length scale in the problem. Such presheath solutions exist, as mentioned previously in Sec. 2, but only for charge-exchange reactions or in the absence of surface electron emission. For the plasma in the MPD thruster, additional ionization by electrons emitted from the surface is important under some conditions. Second, radiative heating of the electrode surface by the plasma has been neglected. This not only contributes to direct heating, but also to increased heating by particle bombardment via increased charged particle number densities at the sheath edge. Radiative transfer in the MPD plasma is a topic for detailed study. Although these two deficiencies can affect the quantitative predictions of the surface temperature, they will not affect the existence of the thermal runaway mechanism and unstable operating regimes found here. The theory presented in this paper together with models of the flowing plasma[22,31,32] provide the designer with a number of tools. In addition to the prediction of erosion rates under a variety of possible operating conditions, the proper cathode length, diameter, material, external cooling conditions, and choice of propellant can be systematically evaluated.

Experimental verification of the predictions of the theory presented in this paper requires simultaneous measurement of the plasma current density, plasma charged particle number density near the electrode, amount of external cooling, and the local surface temperature. Although temperature measurements in low-power steady-state MPD thrusters have been made,[34] lack of knowledge with regard to the local current and number densities makes comparison between this theory and existing measurements unreliable. The measured steady-state cathode surface temperatures in a subscale device for thoriated tungsten range from 2087 to 2281 K for a range of total currents from 500 to 1200 A and mass flows from 46.4 to 150.8 mg/s.[34] This is comparable to the parameter range considered in this paper. Despite the difficulties encountered in nonintrusively measuring plasma number densities and current densities, efforts to measure these variables along with the local surface temperature would be desirable and invaluable. Such local surface temperature measurements, if based on optical pyrometry, must also account for reflection from the radiating MPD plasma. As a final point, it must be mentioned that scanning electron micrographs of used cathodes *always* display evidences of craters from spots and microspots in some regions. These spots can be formed easily during the startup transient or the shutdown transient of the arc. Therefore, future experiments aimed at understanding the diffuse mode limits must exercise great caution to make sure that spots are not formed during arc initiation or shutdown.

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